DIFFERENTIAL CAVITY LENGTH, EXPERIMENTAL CRDS SENSOR APPARATUS*

SMIDC in Association with CAS

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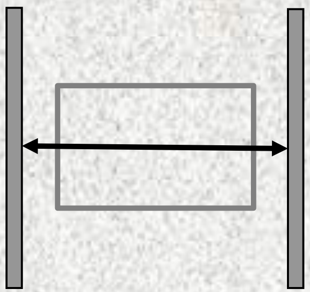
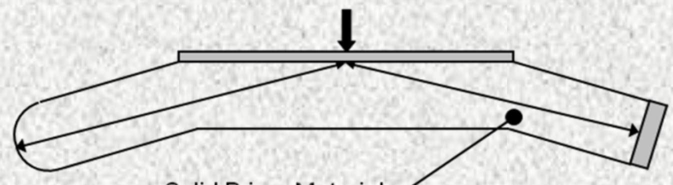
Cavity Ring-Down Spectroscopy (CRDS)



Volume Absorption Form



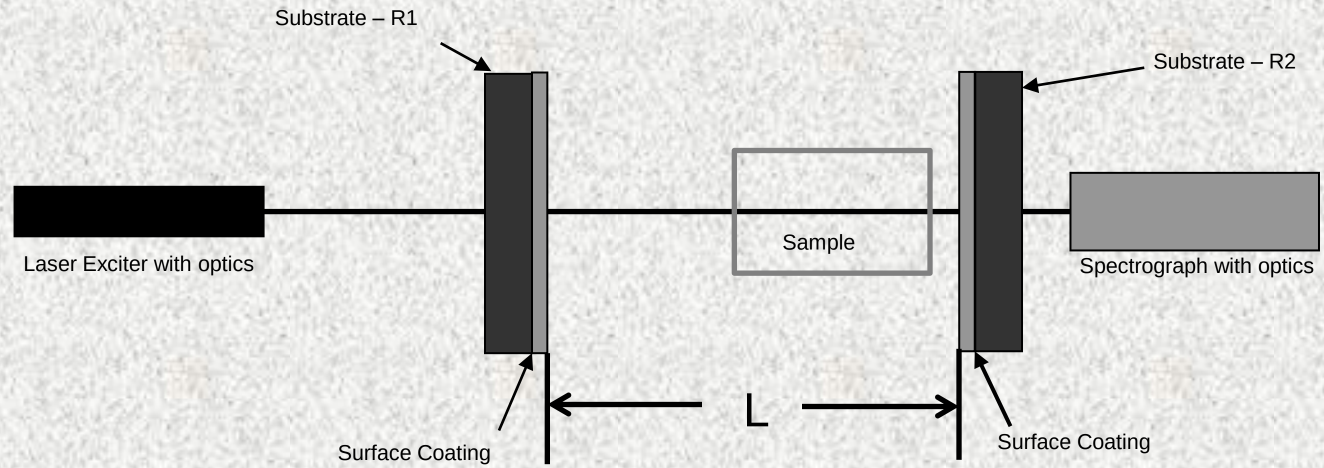
Surface Absorption Form

Cavity Geometry comes in at least two forms

The left is internalized volumetric Format

The right is surface absorption Format

Basic Sensor



Fabry-Perot Cavity

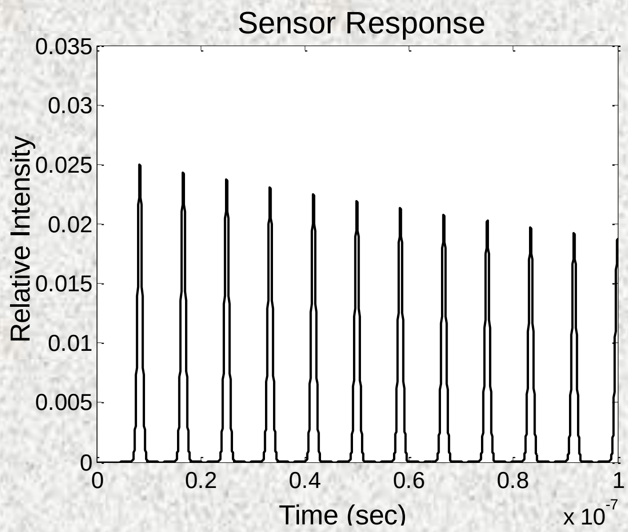
Of interest here is cavity length

Traditional Noise Terms

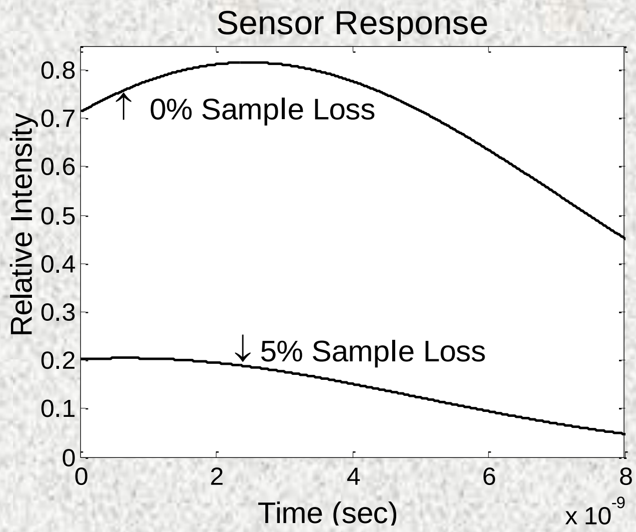
To appreciate the noise issues to follow, a brief review of traditional terms seems in order.

Source Noise	Pre-detector	Post Detector
Electronic	Interference	Photon
Solid State	Multipath	Johnson
Radiation	Solid State	Shot
		G-R
		1/f
		Microphonics
		Electronic

Impact of Cavity Length on Data Stream



“Long” Cavity

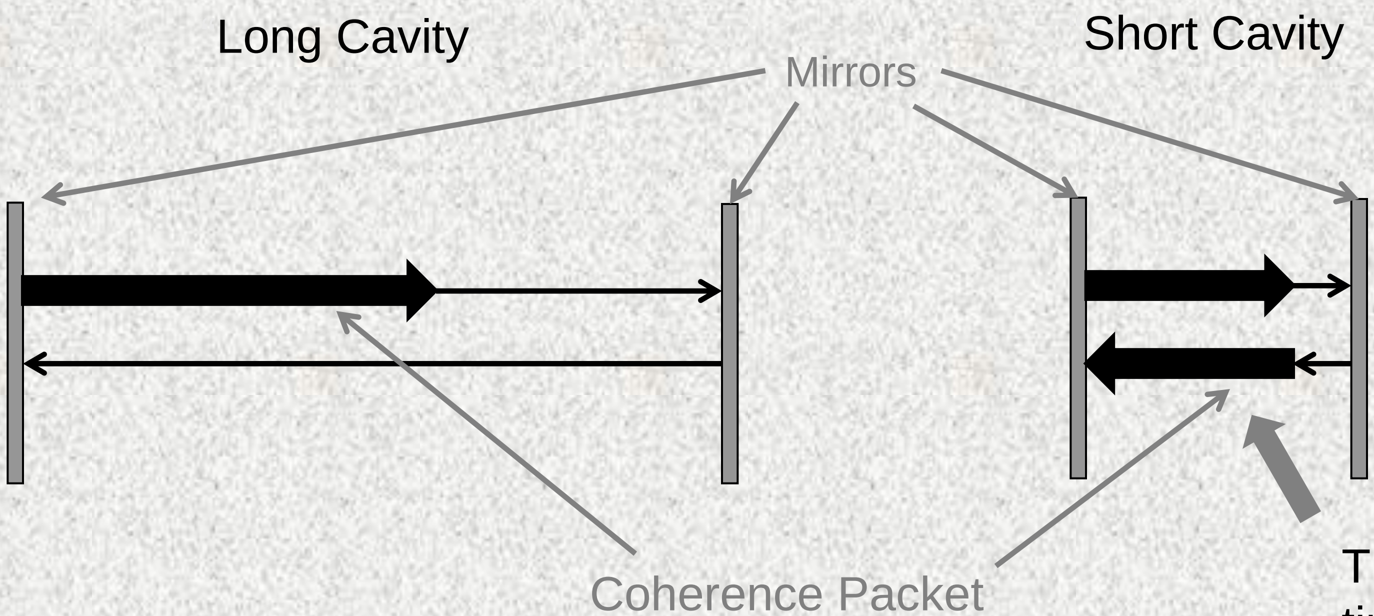


“Short” Cavity

MatLab Modeling of Cavity for Short and Long Cavity

Unusually Short Cavity Inserts a Series of Research Issues

Upon overlap of the spatial and temporally coherent driver pulses, partial coherence develops as a competition between: the long spatial coherence length and the long coherence time associated with the driver, and the cavity length and the loss of coherence due to various optical elements.



This produces time varying fringes, etc. i.e. Speckle

Long and short within context of coherence length

Noise Generation with Same Probability Density Function (PDF) as Speckle

The output a Fabry-Perot cavity, where the driver coherence lengths exceed the cavity length, yields a speckle pattern at the output mirror

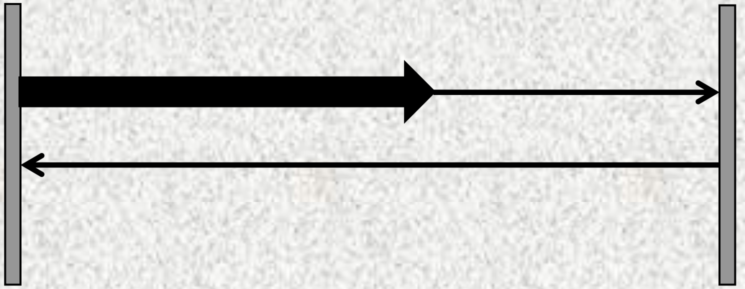
The detector receives that pattern, whose total value fluctuates about a mean (signal) by some variance (Noise Power)

Modeling noise performance of the sensor requires estimates of the PDF, and, noise and signal power

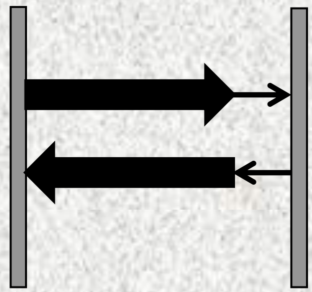
Fortunately all are analytically treatable

Impact of High Power Density on Sample Space

Long Cavity



Short Cavity



Generally, cavity power density is increased by cavity quality factor

As cavity length decreases, energy density increases by volumetric change

Coherence intensity structure increases localized heating

Net effect of all factors is an increase in optical self generated turbulence due to non-uniform beam profile and speckle

Modeling and Experimentation

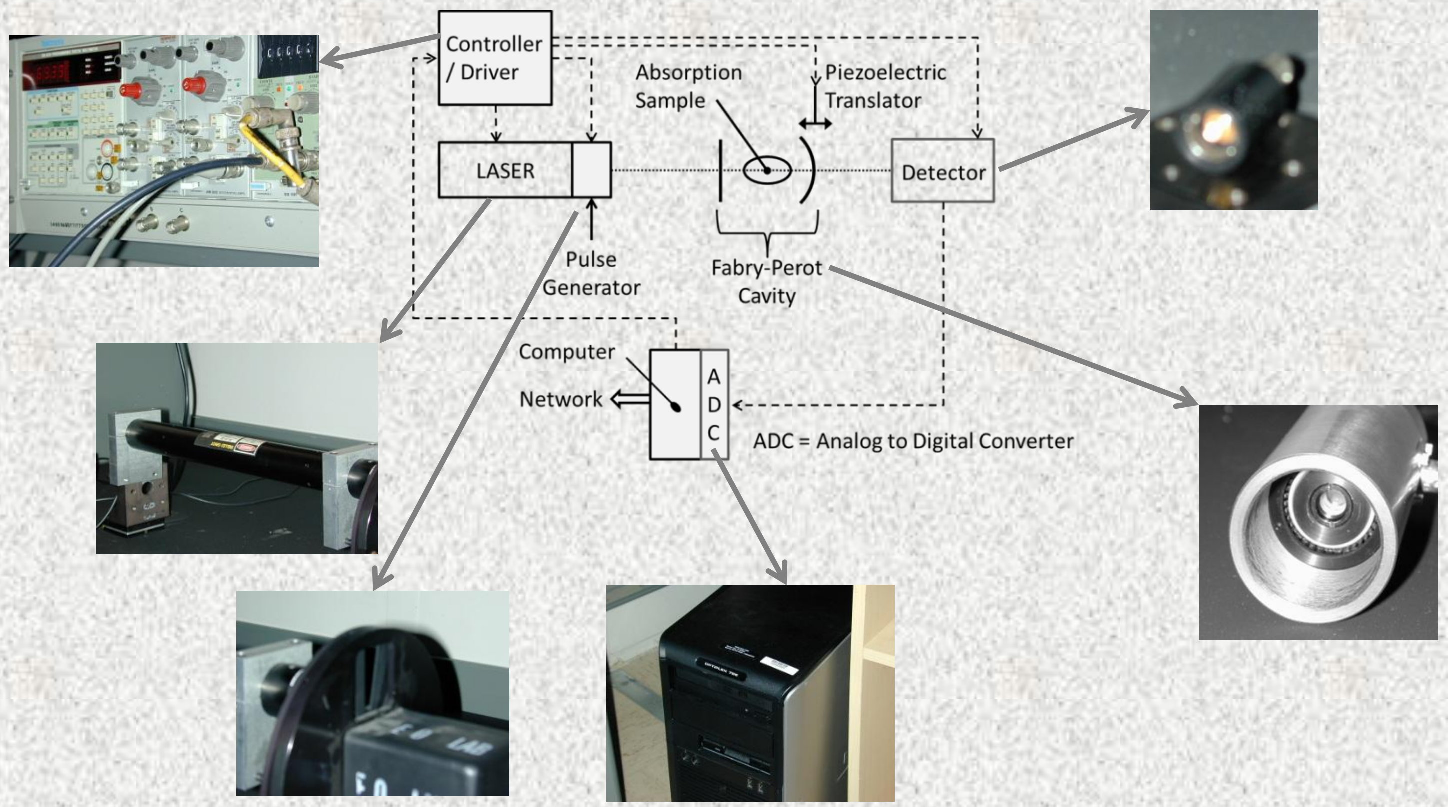
Initial performance modeling indicates a closer look at these two previous un-developed issues.

The research approach is to adapt prior modeling work into the CRDS application and validate

Modeling was accomplished by mathematical closed form and computer simulation with MatLab

Experimentation was accomplished by empirical apparatus in Level 1 Faraday cage using COTS as detailed next

Experimental Configuration



Short Cavity Characterization Apparatus

Results

Modeling and empirical results to date follow, as well as, anticipated future needs.

Speckle-Like Contribution

There are two contributors above and beyond the traditional noise terms for optical detection – The first is a speckle like term that produces noise at the detector plane by impacting spatial coherence.

$$\bar{\mathcal{G}}_i(\nu_u,\nu_v) = (\bar{I_i})^2\left(\bar{\lambda}z_2\right)^2 \iint_{-\infty}^{+\infty}\left|\hat{P}(x,y)\right|^2\left|\hat{P}\left(x-\bar{\lambda}z_2\nu_u, y-\bar{\lambda}z_2\nu_v,\right)\right|^2 dxdy$$

Where

Speckle partial coherence term reduces to retarded wave version of normalized exit pupil of optical description of Fabry-Perot where normalized exit pupil is given as

$$\left|\hat{P}(x,y)\right|^2 = \frac{|P(x,y)|^2}{\iint_{-\infty}^{+\infty}|P(x,y)|^2 dxdy}$$

with

With definitions

$P(x,y)$ is the exit pupil
$\bar{\mathcal{G}}_i(\nu_u,\nu_v)$ is the time average of the power spectral density
ν_u, ν_v are the spatial frequencies in the x,y directions
$\bar{I_i}$ is the incident intensity
$\bar{\lambda}$ is the time average wavelength
$\delta(\nu_u,\nu_v)$ is 2D, unit area impulse function
z_2 is longitudinal distance exit pupil
to point of interest (assumed paraxial)
x,y orthogonal, lateral distances

Effective Apodized Exit Pupil

$\mathrm{TEM}_{m,p}$

$$\mathrm{E}(x,y,z) = E_{m,p} H_m\left[\frac{\sqrt{2}x}{\omega(z)}\right] H_p\left[\frac{\sqrt{2}y}{\omega(z)}\right]\left(\frac{\omega_o}{\omega(z)}\right) \quad e^{-\frac{x^2+y^2}{\omega^2(z)}} e^{-j\left(kz-(1+m+p)\tan^{-1}\left(z/z_o\right)\right)} e^{-j\frac{kr^2}{2R(z)}}$$

Where $H_m[u] = (-1)^m e^{u^2}\frac{d^m\left(e^{-u^2}\right)}{du^m}$ $\mathrm{E}(x,y,z)$ is the electric field

$$\omega^2(z) = \omega_o{}^2\left[1+\left(\frac{z}{z_o}\right)^2\right] \qquad R(z) = z\left[1+\left(\frac{z_o}{z}\right)^2\right] \qquad k = {}^{2\pi}/_{\lambda} \qquad r^2 = x^2+y^2$$

λ is the wavelength

$$|E(x,y,z)|^2 = E_o{}^2\left(\frac{\omega_o{}^2}{\omega^2(z)}\right) e^{-\frac{2r^2}{\omega^2(z)}} \text{ for } \mathrm{TEM}_{0,0}$$

$$P(x,y) = \frac{|E(x,y,z)|^2}{|E(0,0,z)|^2} = e^{-\frac{2r^2}{\omega^2(z)}}$$

For a Fabry-Perot cavity driven by a Transverse Electro-Magnetic (TEM) LASER Source

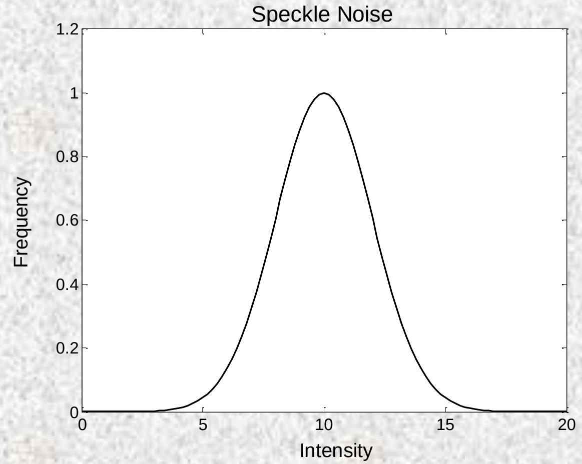
The normalized exit pupil can be derived from a classic Gaussian beam description

Speckle with Apodized Exit Pupil Intrinsic to Cavity

$$\bar{I}_i^{\,2}\left(\frac{2}{\pi}\right)\left[\frac{\left(\bar{\lambda}z_2\right)^2}{\omega^2(z)}\right] \; e^{-\frac{4\left(\bar{\lambda}z_2\right)^2}{\omega^2(z)}} \; e^{-\frac{4\bar{\lambda}^2 z_2^{\,2} \nu_u^{\,2}}{\omega^2(z)}} \; e^{-\frac{4\bar{\lambda}^2 z_2^{\,2} \nu_v^{\,2}}{\omega^2(z)}}$$

Where ν_u and ν_v are the spatial frequencies in the x, y directions respectively

Probability Density Function (PDF) for Detector Noise



As probably can be guessed from above, statistical analysis produces a Gaussian Distribution

Kolmogorov Like Contributions

Thermally driven term due to “Q” enhancement of cavity power density is second unusual noise term

Classic development is lengthy but traces as:

von Karman Spectrum

⇩

Index of Refraction Probability Density

⇩

Riccati Equation

⇩

Noise Probability Density Function

This results in

$$P_I(I) = \frac{1}{2\sqrt{2\pi}\sigma_x I_o e^{-\alpha z}} \quad e^{-\frac{\left(\ln\left(\frac{I}{I_o}\right)-2\bar{\chi}\right)^2}{8\sigma_x^2}}$$

Where

$$\bar{\chi} = \mathrm{Re}\left\{\frac{k_o^2}{2\pi E_o(\vec{r})}\iiint_V \frac{e^{jk_o\left[(z-z')+\frac{|\bar{\rho}-\bar{\rho}'|^2}{2(z-z')}\right]}}{z-z'} n_1(\vec{r}')E_o(\vec{r}')d^3\vec{r}'\right\}$$

$$k_o = {}^{2\pi}/_{\lambda_o}$$

$P_I(I)$ is the log-normal power spectral density
$\vec{r}$ is the spatial vector designating
the observation point with lateral component ρ
and longitudinal component z
$\vec{r}'$, ρ’, z’ are the corresponding
dummy variables of integration
I_o is the incident intensity
λ_o is the time average wavelength
$n_1(\vec{r}')$ is the Index of Refraction Field
described by refractive index structure function
for Kolmogorov Turbulence with
Kolmogorov Power Spectral Densities
$E_o(\vec{r}')$ is electric field where $I = \frac{|E_o|^2}{\eta}$
η = intrinsic impedance

Modified To Accommodate CRDS Application

Embedded in the classic development is the assumption of trivial attentuation in the Optical Path Length (OPL) which is inappropriate for a Fabry-Perot

$$I = I_o e^{-\alpha x}$$

Recasting χ inclusive of traditional photo-absorption assuming constant decay rate results in a reduction of the traditional equation χ yielding a PDF in Intensity of

$$P_I(I) = \frac{1}{2\sqrt{2\pi}\sigma_x I_o e^{-\alpha z}} e^{-\frac{(-\alpha z - 2\bar{\chi})^2}{8{\sigma_x}^2}}$$

$P_I(I)$ *is the intensity PDF*

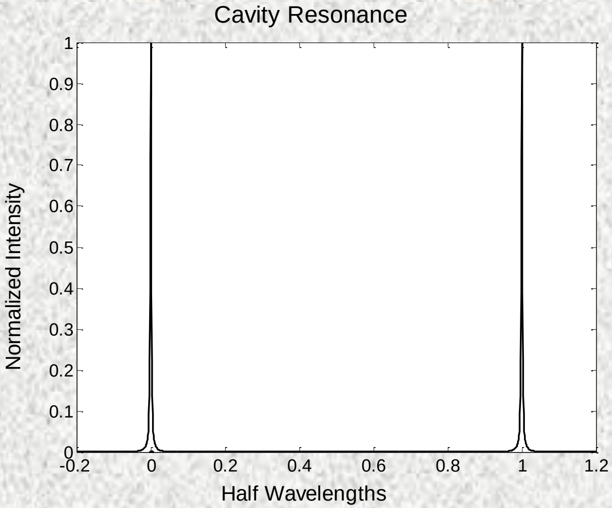
α *is the absorption coefficient*

$\bar{\chi}$ is *the mean (signal)*

${\sigma_x}^2$ *is the variance (noise)*

Cavity Design Implications

Basic system configuration demonstrates instrumentation ability to validate coherence effects of variable cavity length and allows for change of test media. Instrument also allows for validation of coherence mitigation approaches.



It seems likely that some type of coherence elimination element needs to be included in the short cavities.

Future Work: Mitigation Recommendations

Conclusions

A more inclusive noise modeling configuration is presented

Mathematical and Computer simulations have refined the new terms

Empirical results are validating results to date

This additional information may be incorporated into general model development to describing sensor systems